Appendix 1

Equation (7) in the main text is given below:

$$\frac{dY}{dt} = [K_I k_i(1-l) + K_L k_{cl} l][f_s S(t-\tau_d) + D]. \quad (14)$$

Laplace transform of above expression leads to the following:

$$sY(s) - Y(0) = [K_I k_i(1-l) + K_L k_{cl} l][f_s e^{-s\tau_d} S(s) + D(s)],$$
$$sY(s) = [K_I k_i(1-l) + K_L k_{cl} l][f_s e^{-s\tau_d} S(s) + D(s)], \quad (15)$$
$$Y(0) = 0.$$

Let us assume putting which in above equation results in the following expression:

$$sY(s) = [K_I k_i(1-l) + K_L k_{cl} l] f_s e^{-s\tau_d} S(s),$$
$$\frac{Y(s)}{S(s)} = \frac{[K_I k_i(1-l) + K_L k_{cl} l] f_s e^{-s\tau_d}}{s}. \quad (16)$$

Using following approximation in above expression:

$$e^{-s\tau_d} \approx \frac{2-s\tau_d}{2+s\tau_d}, \quad (17)$$

We get:

$$\frac{Y(s)}{S(s)} = \frac{[K_I k_i(1-l) + K_L k_{cl} l] f_s \left(\frac{2-s\tau_d}{2+s\tau_d}\right)}{s}, \frac{Y(s)}{S(s)} = \frac{[K_I k_i(1-l) + K_L k_{cl} l] f_s (2-s\tau_d)}{s(2+s\tau_d)}. \quad (18)$$

Let a step input of magnitude is given to , i.e.,

$$S(s) = \frac{A}{s},$$

Putting which in above expression, we get:

$$Y(s) = \frac{A[K_I k_i(1-l) + K_L k_{cl} l] f_s (2-s\tau_d)}{s^2(2+s\tau_d)}. \quad (19)$$

Using method of partial fractions, we can write:

$$\frac{(2-s\tau_d)}{\tau_d s^2\left(s+\frac{2}{\tau_d}\right)} = \frac{a}{s} + \frac{b}{s^2} + \frac{c}{\tau_d\left(s+\frac{2}{\tau_d}\right)}, \quad (20)$$

Which implies that,

$$\frac{(2-s\tau_d)}{\tau_d s^2\left(s+\frac{2}{\tau_d}\right)} = \frac{a}{s} + \frac{b}{s^2} + \frac{c}{\tau_d\left(s+\frac{2}{\tau_d}\right)},$$
$$2 - s\tau_d = as\tau_d\left(s+\frac{2}{\tau_d}\right) + b\tau_d\left(s+\frac{2}{\tau_d}\right) + cs^2,$$
$$2 - s\tau_d = 2b + (2a + b\tau_d)s + (a\tau_d + c)s^2,$$
$$2b = 2,$$
$$b = 1,$$
$$2a + b\tau_d = -\tau_d,$$
$$a = -\tau_d,$$
$$a\tau_d + c = 0,$$
$$c = -a\tau_d = \tau_d^2.$$

Substituting values of and in Equation (20), provides:

$$\frac{(2-s\tau_d)}{\tau_d s^2\left(s+\frac{2}{\tau_d}\right)} = -\frac{\tau_d}{s} + \frac{1}{s^2} + \frac{\tau_d}{\left(s+\frac{2}{\tau_d}\right)},$$

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Substituting which in Equation (19), gives:

$$Y(s) = [A\{K_I k_i(1-l) + K_L k_{cl} l\} f_s]\left[-\frac{\tau_d}{s} + \frac{1}{s^2} + \frac{\tau_d}{\left(s+\frac{2}{\tau_d}\right)}\right]. \quad (21)$$

Time domain solution for above expression using table of Laplace transform is as follows:

$$Y(t) = [A\{K_I k_i(1-l) + K_L k_{cl} l\} f_s]\left[-\tau_d + t + \tau_d e^{-(2/\tau_d)t}\right], \quad (22)$$

$$Y(0) = 0.$$